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Brincker, Rune; Zhang, Lingmi; Andersen, Palle

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## Output-Only Modal Analysis by Frequency Domain Decomposition

**Rune Brincker**

Department of Building Technology and Structural Engineering, Aalborg University, Sohngaardsholmsvej 57, DK 9000, Aalborg

**Lingmi Zhang**

Institute of Vibration Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, China, P.R.

**Palle Andersen**

Structural Vibration Solutions ApS, NOVI Science Park, Niels Jernes Vej 10, DK 9220 Aalborg East, Denmark

### Abstract

In this paper a new frequency domain technique is introduced for the modal identification of output-only systems, i.e. for the case where the modal parameters must be estimated without knowing the input exciting the system. By its user friendliness the technique is closely related to the classical approach where the modal parameters are estimated by simple peak picking. However, by introducing a decomposition of the spectral density function matrix, the response spectra can be separated into a set of single degree of freedom systems, each corresponding to an individual mode. By using this decomposition technique close modes can be identified with high accuracy even in the case of strong noise contamination of the signals. Also, the technique clearly indicates harmonic components in the response signals.

### 1. Introduction

Modal identification of output-only systems is normally associated with the identification of modal parameters from the natural responses of civil engineering structures, space structures and large mechanical structures. Normally, in these cases the loads are unknown, and thus, the modal identification has to be carried out based on the responses only. Real case examples on some civil engineering structures can be found in Ventura and Horyna [1] or Andersen et al. [2].

The present paper deals with a new way of identifying the modal parameters of a structure from the responses only when the structure is loaded by a broad-banded excitation.

The technique presented in this paper is an extension of the classical frequency domain approach often referred to as the Basic Frequency Domain (BFD) technique, or the Peak Picking technique. The classical approach is based on

simple signal processing using the Discrete Fourier Transform, and is using the fact that well separated modes can be estimated directly from the power spectral density matrix at the peak, Bendat and Piersol [3]. Other implementations of the technique make use of the coherence between channels, Felber [4].

The classical technique gives reasonable estimates of natural frequencies and mode shapes if the modes are well separated. However, in the case of close modes, it can be difficult to detect the close modes, and even in the case where close modes are detected, estimates becomes heavily biased. Further, the frequency estimates are limited by the frequency resolution of the spectral density estimate, and in all cases, damping estimation is uncertain or impossible.

Even though the classical approach has limitations concerning accuracy in the identification process, the classical approach has important advantages when compared to other approaches. It is natural to compare with classical two-stage time domain

approaches such as the Polyreference technique, Vold et al [5], the Ibrahim Time Domain technique, Ibrahim and Milkulcik [6], and the Eigensystem Realization Algorithm, Juang and Papa [7], or to compare with the new one-stage time domain identification techniques known as the Stochastic Subspace Identification algorithms, Van Overschee and De Moor [8]. The main advantages compared to these other techniques is that the classical approach is much more user-friendly, it is faster, simpler to use, and gives the user a "feeling" of the data he or she is dealing with. The fact that the user works directly with the spectral density functions helps the user in figuring out what is structural just by looking at the spectral density functions. This reinforces the user's understanding of the physics and thus provides a valuable basis for a meaningful identification.

The technique presented in this paper is a Frequency Domain Decomposition (FDD) technique. It removes all the disadvantages associated with the classical approach, but keeps the important features of user-friendliness and even improves the physical understanding by dealing directly with the spectral density function. Further, the technique gives a clear indication of harmonic components in the response signals.

In this paper it is shown that taking the Singular Value Decomposition (SVD) of the spectral matrix, the spectral matrix is decomposed into a set of auto spectral density functions, each corresponding to a single degree of freedom (SDOF) system. This result is exact in the case where the loading is white noise, the structure is lightly damped, and when the mode shapes of close modes are geometrically orthogonal. If these assumptions are not satisfied, the decomposition into SDOF systems is approximate, but still the results are significantly more accurate than the results of the classical approach.

## 2. Theoretical background of frequency domain decomposition

The relationship between the unknown inputs  $x(t)$  and the measured responses  $y(t)$  can be expressed as, Bendat & Piersol [9]:

$$G_{yy}(j\omega) = \bar{H}(j\omega)G_{xx}(j\omega)H(j\omega)^T \quad (1)$$

Where  $G_{xx}(j\omega)$  is the  $r \times r$  Power Spectral Density (PSD) matrix of the input,  $r$  is the number of inputs,  $G_{yy}(j\omega)$  is the  $m \times m$  PSD matrix of the responses,  $m$  is the number of responses,  $H(j\omega)$  is the  $m \times r$  Frequency Response Function (FRF) matrix, and " $\bar{\cdot}$ " and superscript  $T$  denote complex conjugate and transpose, respectively.

The FRF can be written in partial fraction, i.e. pole/residue form

$$H(j\omega) = \sum_{k=1}^n \frac{R_k}{j\omega - \lambda_k} + \frac{\bar{R}_k}{j\omega - \bar{\lambda}_k} \quad (2)$$

where  $n$  is the number of modes,  $\lambda_k$  is the pole and  $R_k$  is the residue

$$R_k = \varphi_k \gamma_k^T \quad (3)$$

where  $\varphi_k, \gamma_k$  is the mode shape vector and the modal participation vector, respectively. Suppose the input is white noise, i.e. its PSD is a constant matrix, i.e.  $G_{xx}(j\omega) = C$ , then Equation (1) becomes

$$G_{yy}(j\omega) = \sum_{k=1}^n \sum_{s=1}^n \left[ \frac{R_k}{j\omega - \lambda_k} + \frac{\bar{R}_k}{j\omega - \bar{\lambda}_k} \right] C \left[ \frac{R_s}{j\omega - \lambda_s} + \frac{\bar{R}_s}{j\omega - \bar{\lambda}_s} \right]^H \quad (4)$$

where superscript  $H$  denotes complex conjugate and transpose. Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as follows

$$G_{yy}(j\omega) = \sum_{k=1}^n \frac{A_k}{j\omega - \lambda_k} + \frac{\bar{A}_k}{j\omega - \bar{\lambda}_k} + \frac{B_k}{-j\omega - \lambda_k} + \frac{\bar{B}_k}{-j\omega - \bar{\lambda}_k} \quad (5)$$

where  $A_k$  is the  $k$ th residue matrix of the output PSD. As the output PSD itself the residue matrix is an  $m \times m$  hermitian matrix and is given by

$$A_k = R_k C \left( \sum_{s=1}^n \frac{\bar{R}_s^T}{-\lambda_k - \bar{\lambda}_s} + \frac{R_s^T}{-\lambda_k - \lambda_s} \right) \quad (6)$$

The contribution to the residue from the  $k$ th mode is given by

$$A_k = \frac{R_k \bar{C} R_k^T}{2\alpha_k} \quad (7)$$

where  $\alpha_k$  is minus the real part of the pole  $\lambda_k = -\alpha_k + j\omega_k$ . As it appears, this term becomes dominating when the damping is light, and thus, is case of light damping, the residue becomes proportional to the mode shape vector

$$A_k \propto R_k \bar{C} R_k = \phi_k \gamma_k^T C \gamma_k \phi_k^T = d_k \phi_k \phi_k^T \quad (8)$$

where  $d_k$  is a scalar constant. At a certain frequency  $\omega$  only a limited number of modes will contribute significantly, typically one or two modes. Let this set of modes be denoted by  $Sub(\omega)$ . Thus, in the case of a lightly damped structure, the response spectral density can always be written

$$G_{yy}(j\omega) = \sum_{k \in Sub(\omega)} \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\phi}_k \bar{\phi}_k^T}{j\omega - \bar{\lambda}_k} \quad (9)$$

This is a modal decomposition of the spectral matrix. The expression is similar to the results one would get directly from Equation (1) under the assumption of independent white noise input, i.e. a diagonal spectral input matrix.

PSD  $\hat{G}_{yy}(j\omega)$  known at discrete frequencies  $\omega = \omega_i$  is then decomposed by taking the Singular Value Decomposition (SVD) of the matrix

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^H \quad (10)$$

where the matrix  $U_i = [u_{i1}, u_{i2}, \dots, u_{im}]$  is a unitary matrix holding the singular vectors  $u_{ij}$ , and  $S_i$  is a diagonal matrix holding the scalar singular values  $s_{ij}$ . Near a peak corresponding to the  $k$ th mode in the spectrum this mode or may be a possible close mode will be dominating. If only the  $k$ th mode is dominating there will only be one term in Equation (9). Thus, in this case, the first singular vector  $u_{i1}$  is an estimate of the mode shape

$$\hat{\phi} = u_{i1} \quad (11)$$

and the corresponding singular value is the auto power spectral density function of the corresponding single degree of freedom system, refer to Equation (9). This power spectral density function is identified around the peak by comparing the mode shape estimate  $\hat{\phi}$  with the singular vectors for the frequency lines around the peak. As long as a singular vector is found that has high MAC value with  $\hat{\phi}$  the corresponding singular value belongs to the SDOF density function.

From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping can be obtained. In this paper the piece of the SDOF PSD was taken back to time domain by inverse FFT, and the frequency and the damping was simply estimated from the crossing times and the logarithmic decrement of the corresponding SDOF auto correlation function.

### 3. Identification algorithm

In the Frequency Domain Decomposition (FDD) identification, the first step is to estimate the power spectral density matrix. The estimate of the output

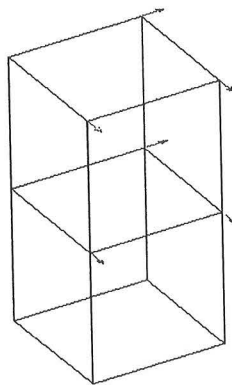


Figure 1. Geometry of 2 storey-building model. Measurement points are indicated by arrows

In the case two modes are dominating, the first singular vector will always be a good estimate of the mode shape of the strongest mode. However, in case the two modes are orthogonal, the first two singular vectors are unbiased estimates of the corresponding mode shape vectors.

In case the two modes are not orthogonal, typically the bias on mode shape estimate of the dominant mode will be small, but the bias on the mode shape estimate of the weak mode will be strong. Thus, one has to estimate the mode shapes for the two close modes at two different frequency lines, one line where the first mode is dominant, and another frequency line where the second mode is dominant.

#### 4. Example, simulation of a 2-storey building

In this example the response of a two-storey building is simulated using a lumped parameter system with 6 degrees of freedom. The measurements are assumed to be taken so that the rigid body motions of the floor slabs can be estimated. The geometry and the measurement points are shown in Figure 1.

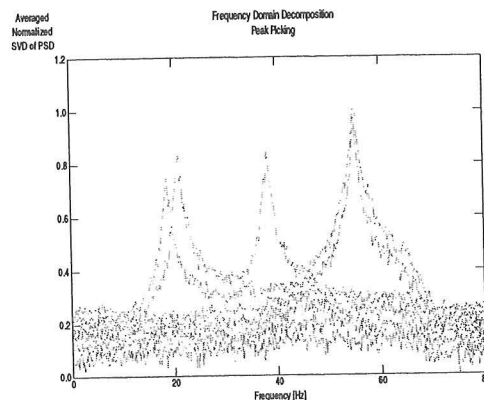


Figure 2. Singular values of the power spectral density matrix of the response

This structure has two sets of close modes. The first two modes are bending modes, and the model was calibrated in such a way, that these two bending modes were close, but not very close. The third mode is a torsion mode, and the fourth and fifth modes are again close bending modes. The model was calibrated in such a way that the fourth and fifth modes were very close, nearly repeated poles.

The response was simulated using a vector ARMA model to ensure that the simulated responses were covariance equivalent, Andersen et al. [10]. The model was loaded by white noise, and the response was analysed using the identification technique introduced above. The simulated time series had a length of 10000 data points and three cases were considered: no noise, 10 % noise and 20 % noise added.

The singular values of the spectral density function matrix are shown in Figure 2. As it appears, the close modes are clearly indicated in this plot. Using the FDD identification procedure described above, the natural frequencies and damping ratios were identified with

high accuracy, see Table 1 for the natural frequencies and Table 2 for the damping values. As it appears, the technique is not sensitive to the noise. Also the mode shape estimates were very close to the exact results. Note especially the mode shapes for the two



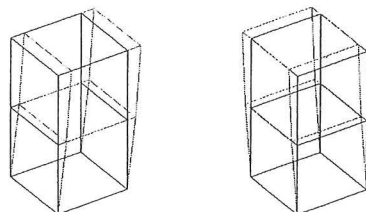


Figure 3. Estimated mode shapes for the first and the second mode (building bending).

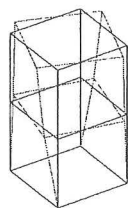


Figure 4. Estimated mode shape for the third mode (building torsion).

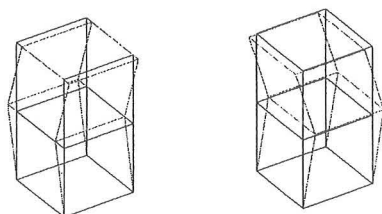


Figure 5. Estimated mode shapes for the fourth and the fifth mode (building bending).

Table 1. Estimated Natural Frequencies

| Exact<br>(Hz) | Without<br>noise<br>(Hz) | 10 %<br>noise<br>(Hz) | 20 %<br>noise<br>(Hz) |
|---------------|--------------------------|-----------------------|-----------------------|
| 18.686        | 18.676                   | 18.661                | 18.665                |
| 21.054        | 20.930                   | 20.927                | 20.938                |
| 38.166        | 38.188                   | 38.188                | 38.206                |
| 55.055        | 55.036                   | 55.011                | 54.999                |
| 55.121        | 55.129                   | 55.133                | 55.125                |

Table 2. Estimated Damping Ratios

| Exact<br>(%) | Without<br>noise<br>(%) | 10 %<br>noise<br>(%) | 20 %<br>noise<br>(%) |
|--------------|-------------------------|----------------------|----------------------|
| 2.13         | 2.22                    | 2.19                 | 2.33                 |
| 1.89         | 1.97                    | 1.98                 | 1.97                 |
| 1.04         | 1.12                    | 1.11                 | 1.13                 |
| 0.72         | 0.61                    | 0.61                 | 0.55                 |
| 0.72         | 0.76                    | 0.76                 | 0.77                 |

nearly repeated modes (the fourth and the fifth) in Figure 5.

## 5. Identification of harmonics, plate loaded by engine

As explained earlier, the frequency domain decomposition (FDD) technique presented in this paper decomposes the spectral density into a set auto spectral density functions each one corresponding to one of the single degree of freedom system representing the corresponding mode. If a harmonic is present, this corresponds a local amplification of the auto spectral density function of all the SDOF systems, i.e. all - or nearly all - of the singular values in the spectral plot will show a peak at the frequency where the harmonic is present. This result holds also in the case of a quasi-stationary harmonic, i.e. in the case of a harmonic with a slowly varying frequency.

Thus, if one observes, that not only the first singular value has a peak at a certain frequency, but most of the other singular values also have a peak at that same frequency, then this is strong indication, that the peak does not represent a structural response, but a harmonic.

If a structural mode is close to the harmonic, the harmonic does not destroy the mode shape estimate. However, one should be careful not using the amplified values of the SDOF bell (amplified by the harmonic) when using the inverse Fourier transform to estimate frequency and damping in the time domain. This will heavily bias the frequency and damping estimate.

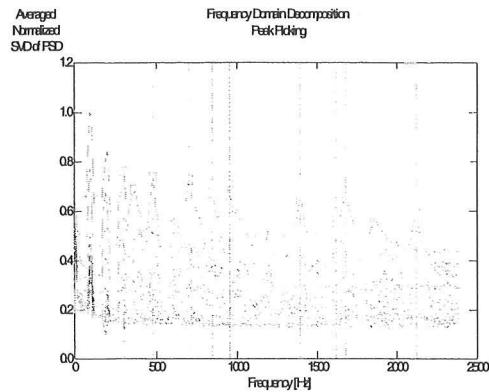


Figure 6. Decomposed spectral density for the plate loaded by rotating engine only

The example presented here is the response of a rectangular plate to the oscillations of an engine. The engine rotates with a quasi-stationary speed corresponding to a frequency of about 100 Hz, but it also excites the plate by a broad-banded loading due to noise generated by the bearing etc. The out-of-plane deformations of the plate were measured using 16 accelerometers, and the signals were acquired by a Brüel & Kjær PULSE™ multi-analyser system where time recording takes place by throughput to disk. The response signals were sampled at a sampling rate of 8192 data points per second, and 81920 data points were acquired corresponding to a total measurement time of 10 seconds. For the modal analysis the raw time data were exported into the ARTEMIS software from SVS using the Universal File format. The test is described in detail in Møller et al. [15].

The decomposed spectrum shown in figure 6 clearly indicates that the first 2 peaks are harmonics (all singular values are influenced). The 3<sup>rd</sup> harmonic is not so clearly indicated by the singular values.

However, it can also be identified as a harmonic from the fact that it is an over-harmonic to the first and that the mode shape estimated from the peak is nearly the same as the mode shape estimated for the first structural mode, see figure 7.

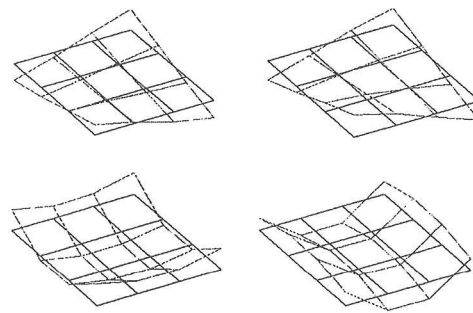


Figure 7. Upper left: Harmonic 304 Hz. Upper right: 1<sup>st</sup> mode 368 Hz. Lower left: 2<sup>nd</sup> mode 480 Hz. Lower right: 3<sup>rd</sup> mode 704 Hz.

## Conclusions

In this paper a new frequency domain identification technique denoted Frequency Domain Decomposition (FDD) has been introduced.

The technique is based on decomposing the power spectral density function matrix using the Singular Value Decomposition. It has been shown that this decomposes the spectral response into a set of single degree of freedom systems, each corresponding to one individual mode.

The technique has been illustrated on a simulation example with noise and close modes. The results clearly indicate that the present technique is able to estimate close modes with high accuracy and that the technique is not sensitive to noise.

In the case of close modes that are not orthogonal, the mode shape of the dominant mode is still well estimated. However, if the other mode is not dominating a any frequency, other ways of estimating the mode shape for such a mode must be introduced.

The technique has been applied successfully to several civil engineering cases, Brincker et al. [11], [12] and to several cases of identification in mechanical engineering where the structure was loaded by rotating machinery, Brincker et al. [13], Møller et al. [14] and [15].

The technique clearly indicates the presence of harmonics in the response signal, i.e. without further indication the user directly separates harmonic peaks from structural response peaks.

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